

# C.U.SHAH UNIVERSITY

## Winter Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1

Branch: Diploma (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  - (2) Instructions written on main answer book are strictly to be obeyed.
  - (3) Draw neat diagrams and figures (if necessary) at right places.
  - (4) Assume suitable data if needed.
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Q-1

Attempt the following questions:

(14)

- a) The distance between the points (1, 3) and (0, -4) is \_\_\_\_\_.  
(A) 50 (B)  $5\sqrt{2}$  (C)  $2\sqrt{5}$  (D) None of these
- b) If (3, 8), (4, 2) and (-1, 5) are the vertices of a triangle, then the co ordinates of its centroid is \_\_\_\_\_.  
(A) (2, -5) (B) (-2, 5) (C) (2, 5) (D) None of these
- c) Slope of the line  $2x - 3y + 4 = 0$  is \_\_\_\_\_.  
(A)  $-2/3$  (B)  $-3/2$  (C)  $3/2$  (D)  $2/3$
- d) Radius of the circle  $x^2 + y^2 = 7$  is \_\_\_\_\_.  
(A) 7 (B)  $\sqrt{7}$  (C)  $\frac{7}{2}$  (D) None of these
- e)  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x =$  \_\_\_\_\_.  
(A)  $e^2$  (B)  $e$  (C)  $e^{1/2}$  (D) None of these
- f)  $\lim_{x \rightarrow \infty} x \left[ \sqrt[3]{7} - 1 \right] =$  \_\_\_\_\_.  
(A) 0 (B)  $e^7$  (C)  $\log_e 7$  (D) 1
- g)  $\lim_{x \rightarrow 1} \frac{x^2 + 3x + 4}{x + 2} =$  \_\_\_\_\_.  
(A)  $8/3$  (B)  $3/8$  (C) 2 (D) 3
- h)  $\frac{d(a^x)}{dx} =$  \_\_\_\_\_.  
(A)  $a^x \log_e a$  (B)  $\log_e a$  (C)  $x^a \log_e a$  (D)  $a^x \log_e x$
- i)  $\frac{d(x^2 + 2x + 7)}{dx} =$  \_\_\_\_\_.  
(A)  $2x$  (B)  $2x + 1$  (C)  $2x + 2$  (D) None of these



- j)  $\frac{d(\log \cos x)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $\cot x$  (B)  $-\tan x$  (C)  $\operatorname{cosec} x$  (D)  $\sec x$
- k)  $\frac{d(\tan^{-1} x + \cot^{-1} x)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $-1$  (B)  $0$  (C)  $1$  (D) None of these
- l)  $\int \sec^2 x \, dx = \underline{\hspace{2cm}}$   
 (A)  $\tan x + c$  (B)  $\cot x + c$  (C)  $\sec x \tan x + c$  (D)  $-\operatorname{cosec} x \cot x + c$
- m)  $\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \underline{\hspace{2cm}}$   
 (A)  $\log \left| x + \sqrt{x^2 + a^2} \right| + c$  (B)  $\log \left| x + \sqrt{x^2 - a^2} \right| + c$  (C)  $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$   
 (D)  $\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$
- n)  $\int_0^1 \frac{2}{1+x^2} \, dx = \underline{\hspace{2cm}}$   
 (A)  $\pi$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{2}$  (D) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

- a) In which ratio Y – axis divides line segment joining points (1, 2) and (2, 1)? Find co ordinates of division point. (5)
- b) Find the equation of straight line passing through (3, 4) and parallel to line  $\frac{x}{2} + \frac{y}{2} = 1$ . (5)
- c) If  $f(x) = \frac{1-x}{1+x}$  then prove that  $f(x) + f\left(\frac{1}{x}\right) = 0$ . (4)

**Q-3 Attempt all questions (14)**

- a) Prove that  $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \frac{\sqrt{35}}{5}$ . (5)
- b) Evaluate:  $\lim_{\theta \rightarrow 0} \frac{\operatorname{cosec} \theta - \cot \theta}{\theta}$  (5)
- c) A(-1, 3), B(-1, x) and C(4, 3) are vertices of a triangle. If  $m\angle B = 90^\circ$  then find value of x. (4)

**Q-4 Attempt all questions (14)**

- a) Find centre and radius of the circle  $36x^2 + 36y^2 + 24x - 36y - 23 = 0$ . (5)
- b) Find derivative of  $f(x) = e^x$  using definition. (5)
- c) Find  $\frac{dy}{dx}$  if  $y = e^x \sin x \cos x$ . (4)

**Q-5 Attempt all questions (14)**

- a) Evaluate:  $\lim_{n \rightarrow \infty} 4 \left[ \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^4} \right]$  (5)



b) The equation of motion of a particle is  $S = 2t^3 + 3t^2 - 12t + 5$ . (5)  
(i) Find velocity at  $t = 0$ . (ii) Find acceleration at  $t = 1$ .

c) Find  $\frac{dy}{dx}$  if  $y = (\sin x)^{\tan x}$ . (4)

**Q-6**

**Attempt all questions**

**(14)**

a) Evaluate:  $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$  (5)

b) Find  $\frac{dy}{dx}$  if  $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ . (5)

c) Evaluate:  $\int \frac{(1-3x)^2}{x^3} dx$  (4)

**Q-7**

**Attempt all questions**

**(14)**

a) Find  $\frac{dy}{dx}$  if  $y = \log \left[ x + \sqrt{x^2 + a^2} \right]$ . (5)

b) If  $\frac{dy}{dx} = 4x^2 + 6x - 1$  and  $y = 5$  when  $x = 2$ , represent  $y$  as a function of  $x$ . (5)

c) Evaluate:  $\int x e^x dx$  (4)

**Q-8**

**Attempt all questions**

**(14)**

a) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \cos ecx} dx = \frac{\pi}{4}$ . (5)

b) Find the area of the circle  $x^2 + y^2 = r^2$ . (5)

c) Find the equation of a circle passing through point  $(-7, 1)$  and centre  $(-4, -3)$ . (4)

